Theoretical foundations of fuzzy logic

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Fuzzy logic

- - What is fuzzy logic?
- - Basic concepts
  - Crisp vs. Fuzzy sets
  - Fuzzy logic
  - Fuzzy rules
  - Linguistic variables
  - Modus ponens for fuzzy logic
- - Application areas
- - Pros and Cons
- - References

What is fuzzy logic?

- Fuzzy logic theory was developed by **Lofti A. Zadeh** in the 60‘s and is based on the theory of ***fuzzy sets***.

- The membership of an element is not strickly false or true({0,1}) like in boolean logic, but rather gradual.

- The ***degree of membership*** of an element in fuzzy logic can be *any real number in the interval [0,1]*.

Aims of fuzzy logic

- deal with the vagueness and imprecision of many real-world problems.

- to simulate human reasoning and its ability of decision making based on not so precise information.

- to model systems that have to process some kind of vague terms like *old, young, tall, high, very, extremely, not so much*, etc..

What is fuzzy logic?

Some other important characteristics of fuzzy logic as outlined by Zadeh (1992).

- Crisp sets as a particular case of fuzzy sets, where [0,1] is restricted to {0,1}.
- Knowledge is interpreted as a collection of elastic, fuzzy constraints on a collection of variables.
- Inference is viewed as a process of propagation of elastic constraints.
- Any logical system can be "fuzzified."

What is fuzzy logic?

...or one could briefly define fuzzy logic quoting L.A. Zadeh:

Fuzzy logic is ‘computing with words.’

Fuzzy sets

Some formal definition of fuzzy sets

Given a collection of objects U, a fuzzy set A in U is defined as a set of ordered pairs

$$A \equiv \{<x,\mu_A(x)> | x \in U\}$$

where

$$\mu_A(x)$$

is called the membership function for the set of all objects x in U, where to each x is related a real number(membership grade) in the closed interval [0,1].

Temperatures as crisp sets

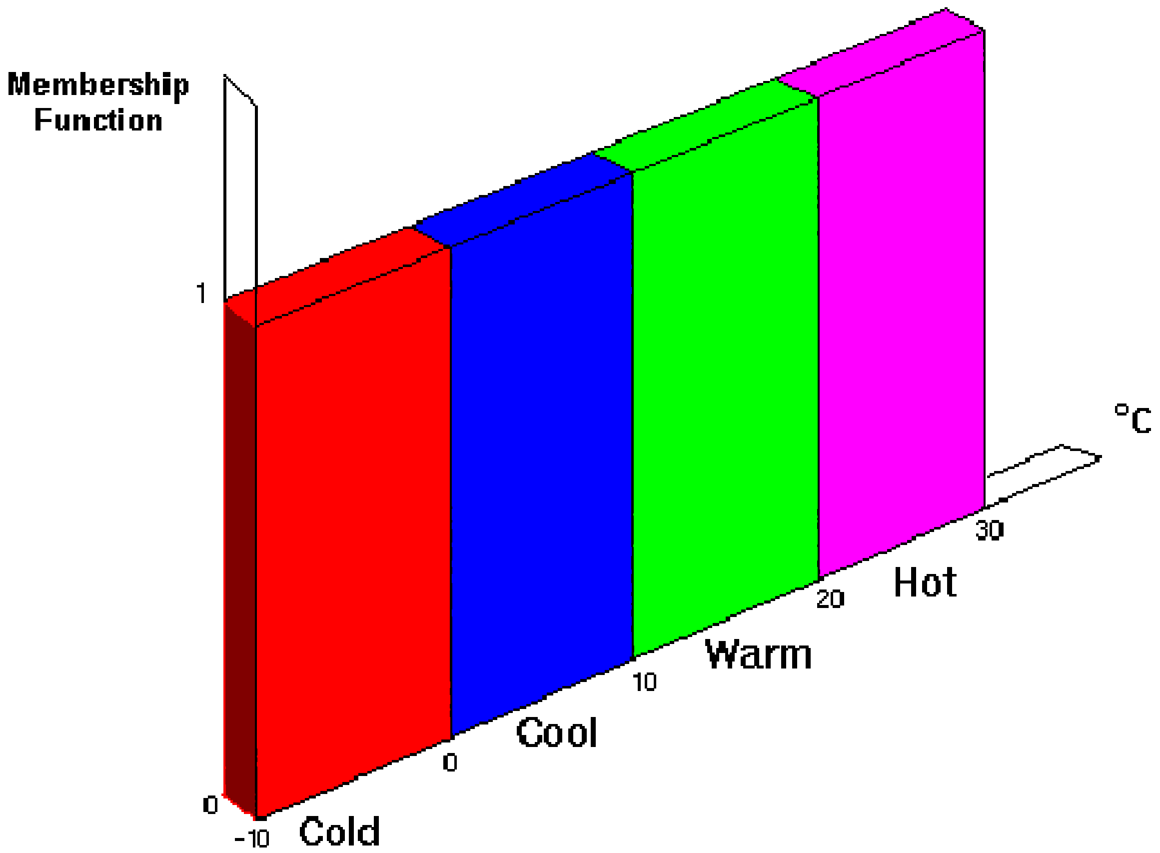


Fig. 1 : Bivalent Sets to Characterize the Temp. of a room.

Source: http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol2/jp6/article2.html

...and as fuzzy sets

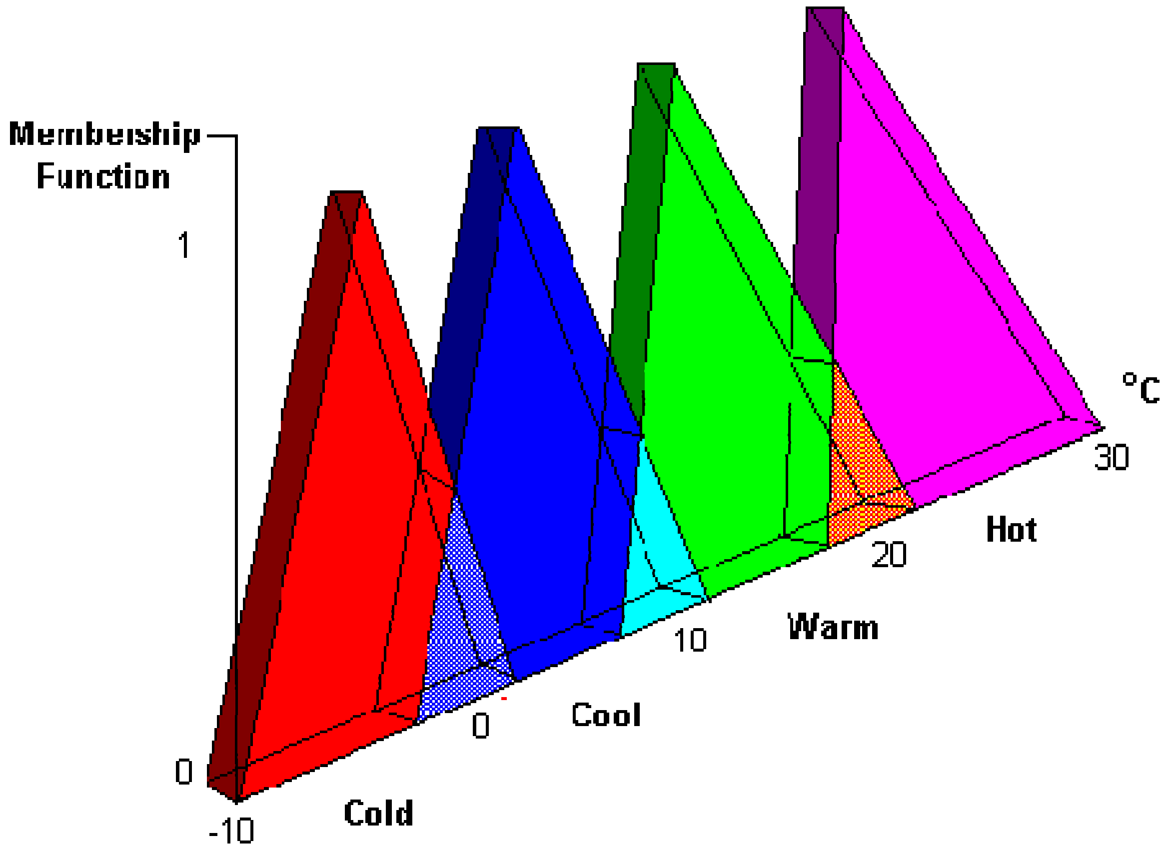


Fig. 2 - Fuzzy Sets to characterize the Temp. of a room.

Source: http://www.doc.ic.ac.uk/~nd/surprise_96/journal/vol2/jp6/article2.html

Other examples...

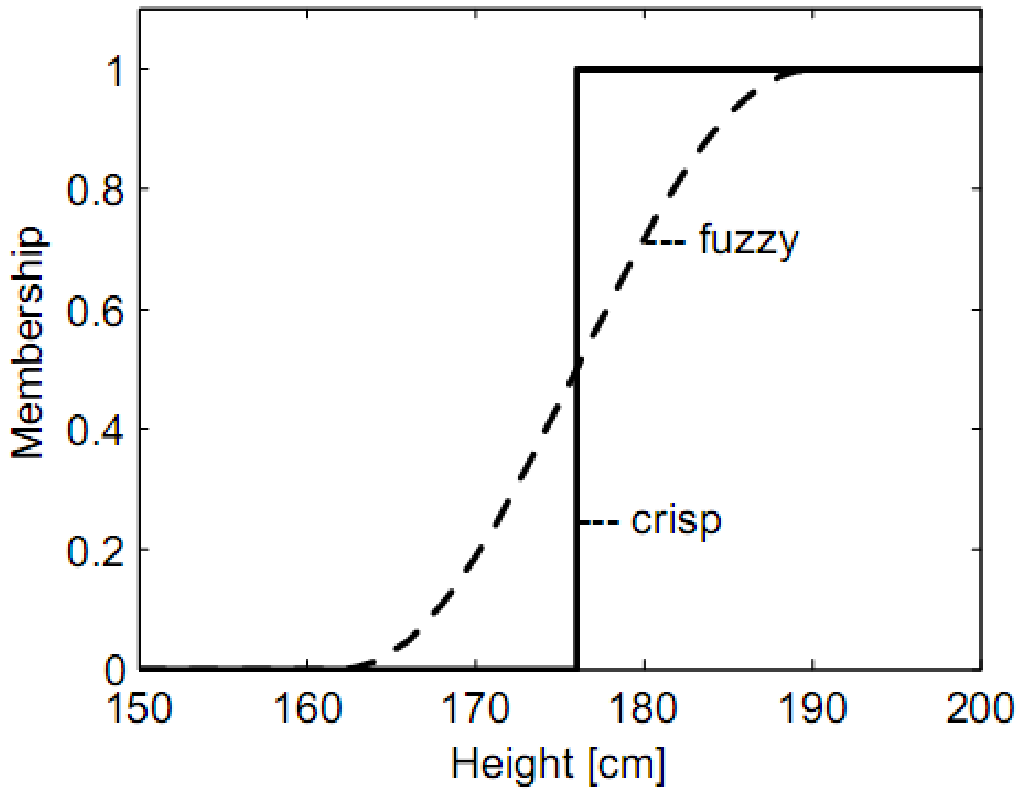


Fig. 3 Two definitions of the set of "tall men", a crisp set and a fuzzy set.

Membership functions

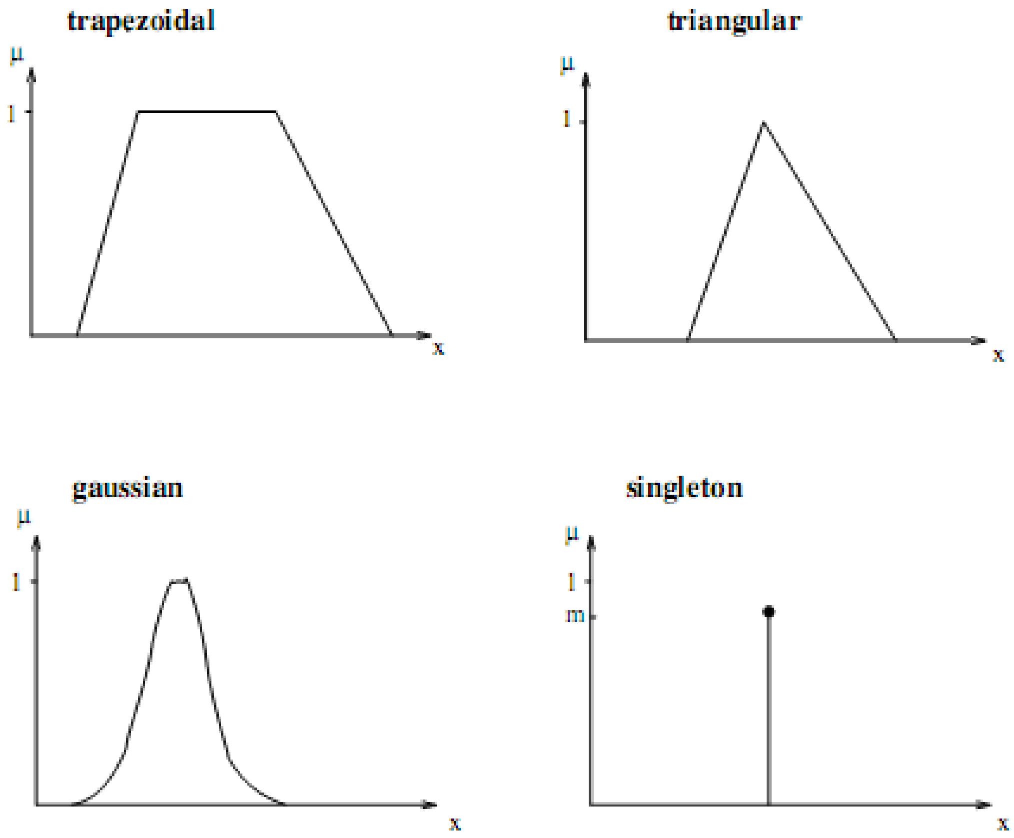


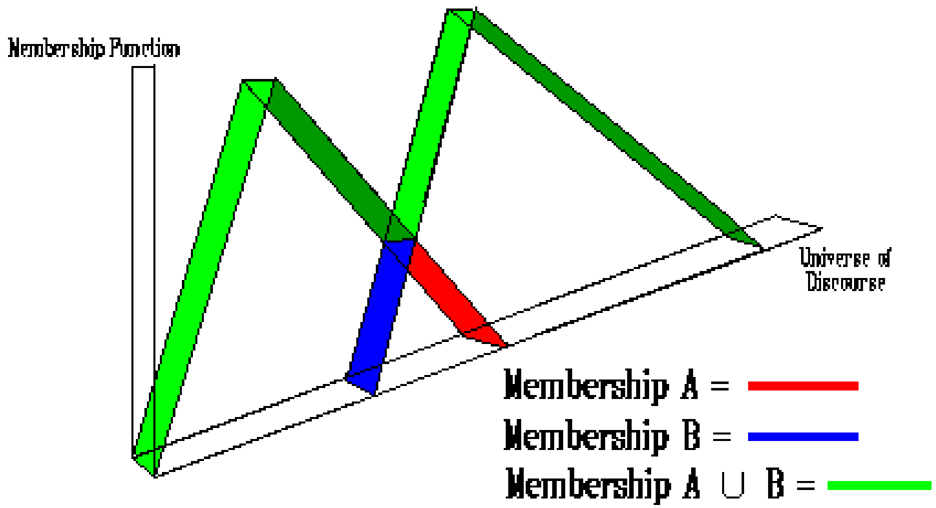
Fig. 4 – Four commonly used membership functions

Basic operations on fuzzy sets

Union:

The membership function of the union of two fuzzy sets A and B with membership functions μA and μB respectively is defined as the maximum of the two individual membership functions:

$$\mu_{A \cup B}(x) := \max\{\mu_A(x), \mu_B(x)\}$$



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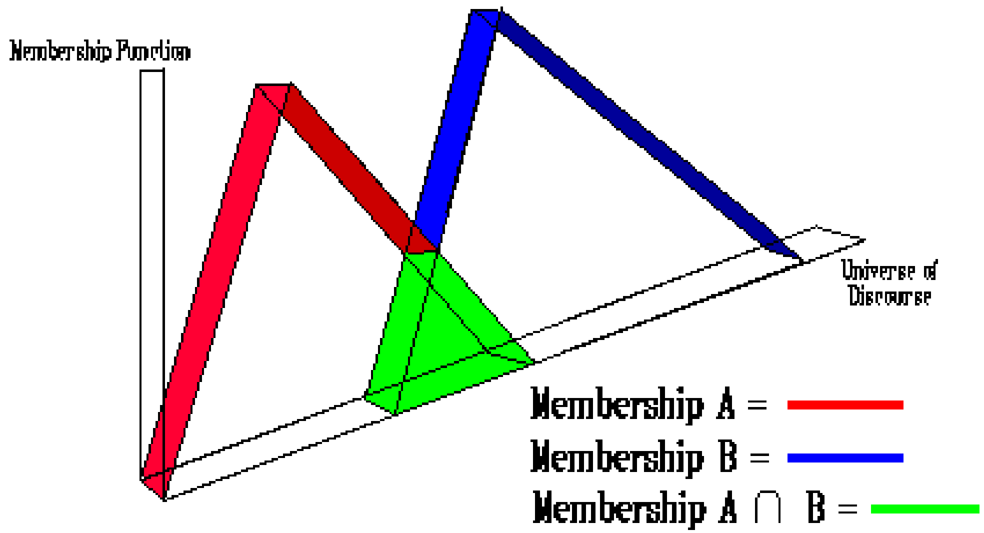
Basic operations on fuzzy sets

Intersection:

The membership function of the intersection of two fuzzy sets A and B with membership functions μA and μB respectively is defined as the minimum of the two individual membership functions:

$$\mu_{A \cap B}(x) := \min\{\mu_A(x), \mu_B(x)\}$$



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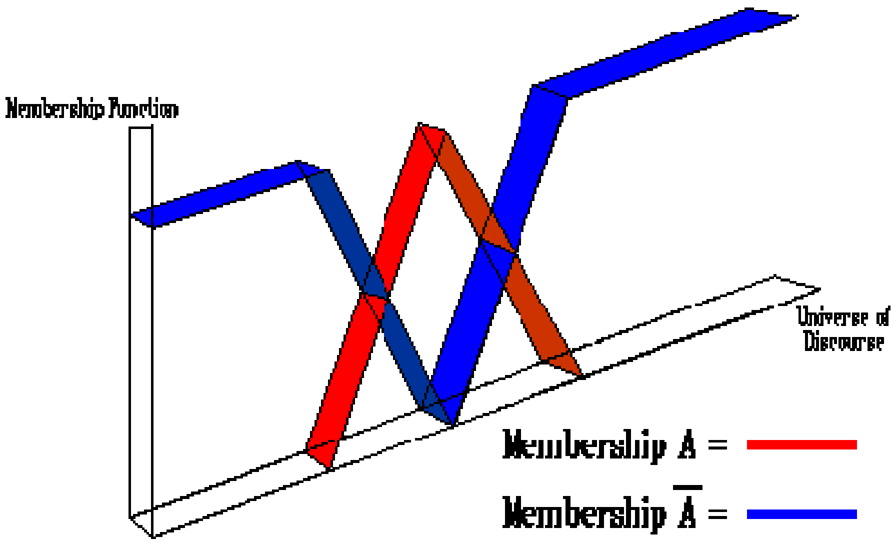
Basic operations on fuzzy sets

Complement:

The membership function of the complement of a Fuzzy set A with membership function is defined as

$$\mu_{\neg A}(x) := 1 - \mu_A(x)$$



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Fuzzy logic operations

The fuzzy set operations of union, intersection and complement correspond to the logical operations disjuntion, conjunction and negation, respectively.

- Disjunction (OR): $\mu A \vee B(x) := \max\{\mu A(x), \mu B(x)\}$
- Conjunction (AND): $\mu A \wedge B(x) := \min\{\mu A(x), \mu B(x)\}$
- Complement(NOT): $\mu \neg A(x) := 1 - \mu A(x)$

Fuzzy sets operations - example

$\mu_{young}(John) = 0.5$ **AND** $\mu_{tall}(John) = 0.8$

then

$\mu_{youngANDtall}(John) = \min(\mu_{young}(John), \mu_{tall}(John))=$

$$0.5 \wedge 0.8 = \min(0.5, 0.8) = 0.5$$

Linguistic variables

Concept:

„By a linguistic variable we mean a variable whose values are words or sentences in a natural or artifical language. For example, *Age* is a linguistic variable if its values are linguistic rather than numerical, i.e., *young*, *not young*, *very young*, *quite young*, *old*, *not very old* and *not very young*, etc.[...]"

(Zadeh, 1975)

Linguistic variables

- The name of a linguistic variable is its label.
- The set of values that it can take is called its term set.
- Each value in the term set is a linguistic value or term defined over an universe.
- In summary: A linguistic variable takes a linguistic value, which is a fuzzy set defined on the universe.

Linguistic variables

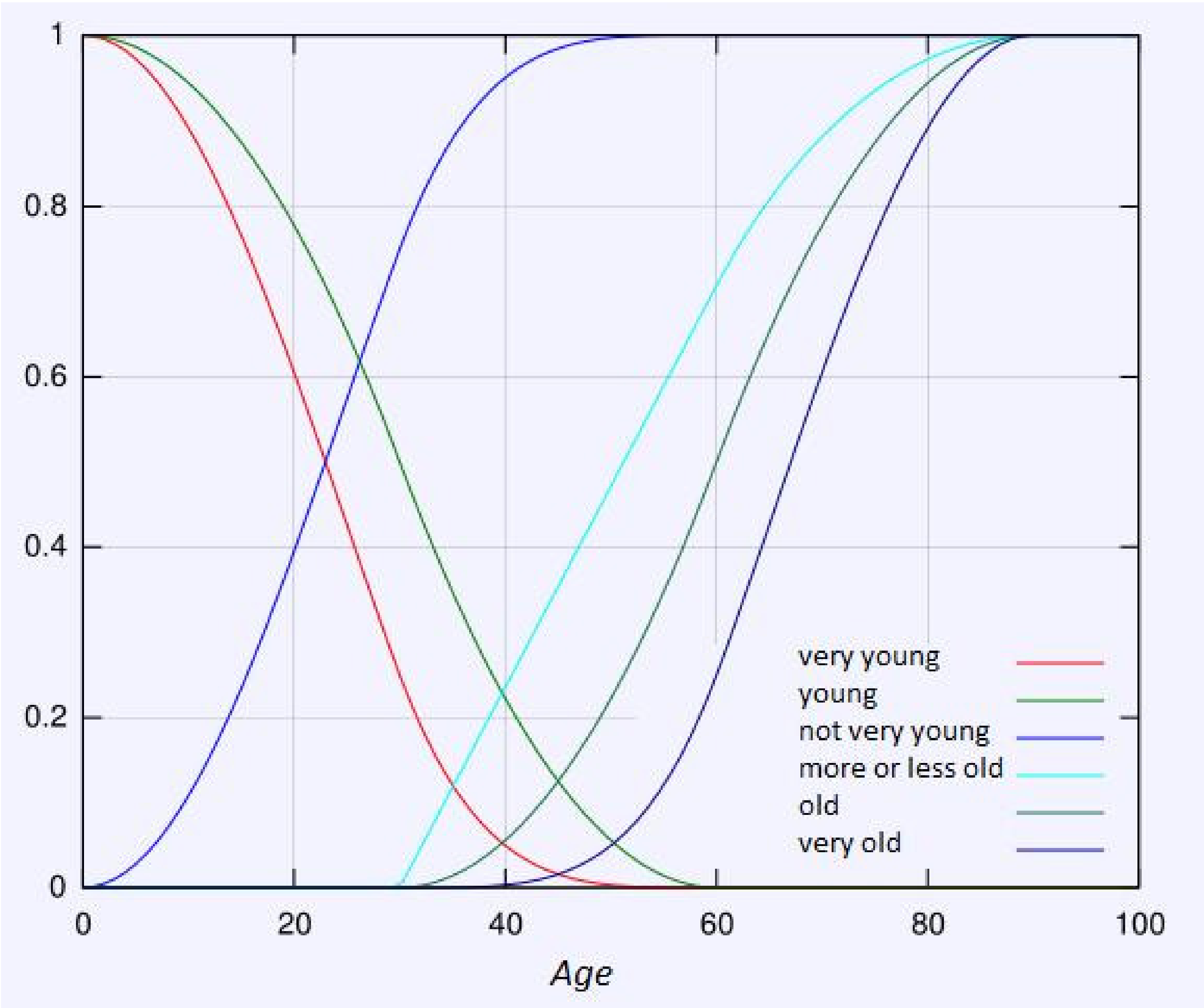
Example:

Let x be a linguistic variable labelled *'Age'*. Its term set *T* could be defined as

T (age)= {very young, young, not very young, more or less old, old}

Each term is defined on the universe, for example the integers from 0 to 100 years.

Example:
A not-linear fuzzy function for *Age*



source: http://de.wikipedia.org/wiki/Datei:Fuzzy-alter.svg

Classical modus ponens

Recalling the classical logic modus ponens:

$$\frac{\begin{array}{l} A \rightarrow B \\ A \end{array}}{B}$$

Fuzzy modus ponens

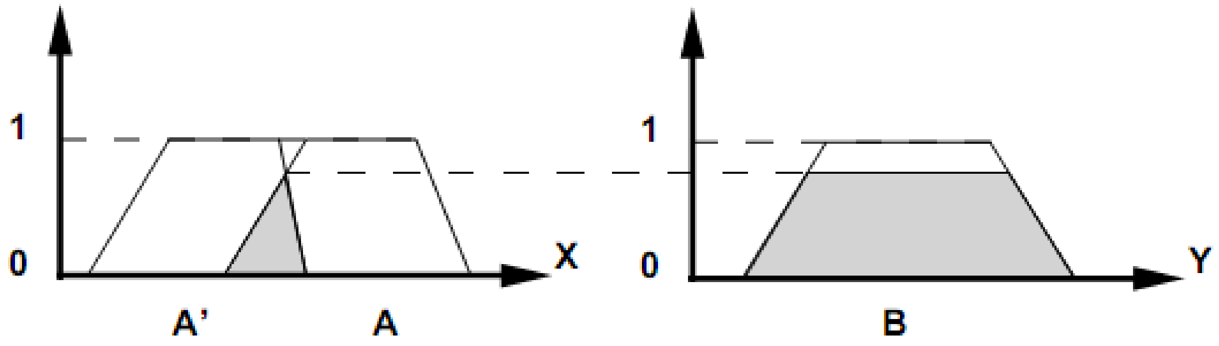
The *Generalized Modus Ponens (GMP)* to fuzzy logic is the core of fuzzy reasoning. Consider the argument:

$$\frac{\begin{array}{l} A \rightarrow B \\ A' \end{array}}{B'}$$

Let A and A‘ be fuzzy sets defined on *X*, and B a fuzzy set defined on *Y*. Then, B‘ is given by

B‘ = A‘ o (A → B)

Fuzzy modus ponens



The interaction of A and A' determines the influence of B in the conclusion

Fuzzy modus ponens

The premise A' is slightly different from A and thus the conclusion B' is slightly different from B.

$$\frac{\begin{array}{l} A \longrightarrow B \\ A' \end{array}}{B'}$$

For instance, given the rule *'if x is high, then y is low'*; if x in fact is *'very high'* , we would like to conclude that y is *'very low'* .

$$\frac{\begin{array}{l} High \rightarrow Low \\ Very\ high \end{array}}{Very\ low}$$

Example GMP

- Given the rule:

  ‘if altitude is ***high***, then oxygen is ***low***’ ,

- a fuzzy set HIGH of altitude ranges:

  HIGH = *{<0,0>,<1000,0.25>,<2000,0.5>,<3000,0.75>, <4000,1>}*

- and a fuzzy set LOW of percentages of oxygen content:

  LOW = *{<0,1>,<25,0.75>,<50,0.5>,<75,0.25>,<100,0>}*

  We construct the Relation R, where each element R_{xy} *is the evaluation of* $\mu_{High}(x) \leq \mu_{Low}(y)$

R =

	1	.75	.5	.25	0
0	1	1	1	1	1
.25	1	1	1	1	0
.5	1	1	1	0	0
.75	1	1	0	0	0
1	1	0	0	0	0

Example GMP

Assuming altitude is *High*, we find by modus ponens

$$\frac{A \longrightarrow B \quad A'}{B'}$$

$$\begin{aligned} \mu^t &= \mu^t_{High} \circ \mathbf{R} \\ &= \begin{pmatrix} 0 & .25 & .5 & .75 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & .75 & .5 & .25 & 0 \end{pmatrix} \end{aligned}$$

$$\frac{High \longrightarrow Low \quad Very\ high}{Very\ low}$$

As expected, the result is identical to μ_{Low}

Example GMP

Assume instead altitude is Very High,

$\mu^t_{VeryHigh}$ = (0 .06 .25 .56 1) , the **square of μ_{High}**, modus ponens yields

$$\mu^t = \mu^t_{VeryHigh} \circ \mathbf{R}$$

$$= \begin{pmatrix} 0 & .06 & .25 & .56 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & .56 & .25 & .06 & 0 \end{pmatrix}$$

$$\frac{\begin{array}{l} High \longrightarrow Low \\ Very\ high \end{array}}{Very\ low}$$

that is, the result is identical to the square of μ_{Low}

Application areas *(summarily)*

- **Digital image processing**, such as edge detection
- Washing machines and other home appliances
- Video game artificial intelligence
- Simplified control of **robots** *(Hirota, Fuji Electric, Toshiba, Omron)*
- Substitution of an expert for the **assessment of stock exchange activities** *(Yamaichi, Hitachi)*
- Efficient and stable control of **car-engines** *(Nissan)*
- **Medicine technology:** cancer diagnosis *(Kawasaki Medical School)*
- Combination of Fuzzy Logic and **Neural Nets** *(Matsushita)*
- Recognition of handwritten symbols with **pocket computers** *(Sony)*

Source: http://www.esru.strath.ac.uk/Reference/concepts/fuzzy/fuzzy_appl.10.htm

Advantages of fuzzy systems

- Robust approach to solve many real-world problems.
- Employable in very complex systems, when there is no simple mathematical model for highly nonlinear processes.
- Hence, low computational costs and ease at using it in embedded systems.
- Expert knowledge in complex systems can be formulated in ordinary language.

Disadvantages

- The number of rules can grow exponentially inverse with the accuracy level. Undesirable high complexity and rule-chaining problem.(Castro, 1999)

- The rules and the membership function for (*imprecise*) data must be (*accurately*) known and defined.

- Must be combined with an adaptive system (such as neural networks) if some heuristics is desired.

References

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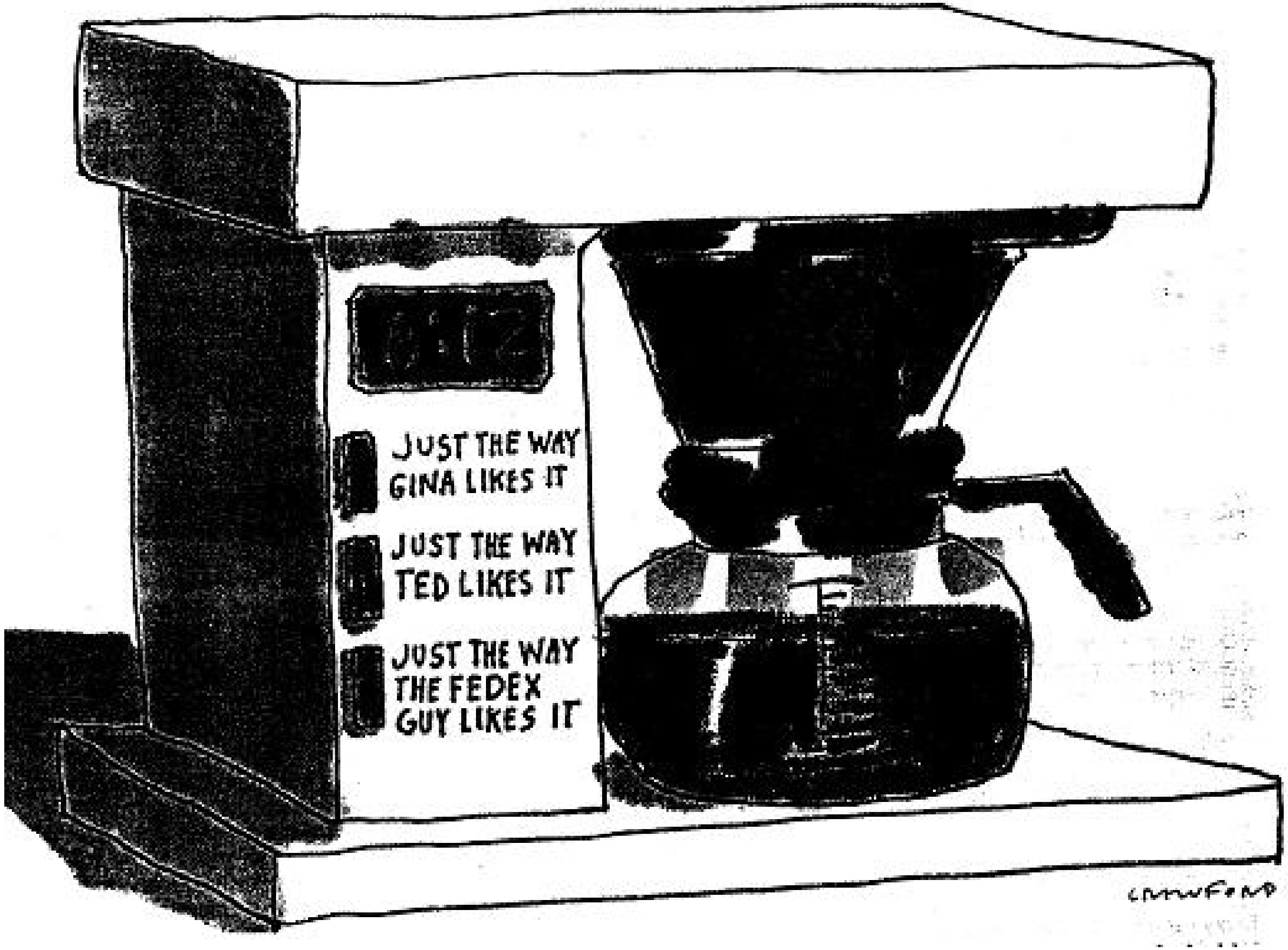

Scientific American Magazine – July 1993 – p.77

QUESTIONS?