

# Statistical Machine Translation

-language models (cont.)-

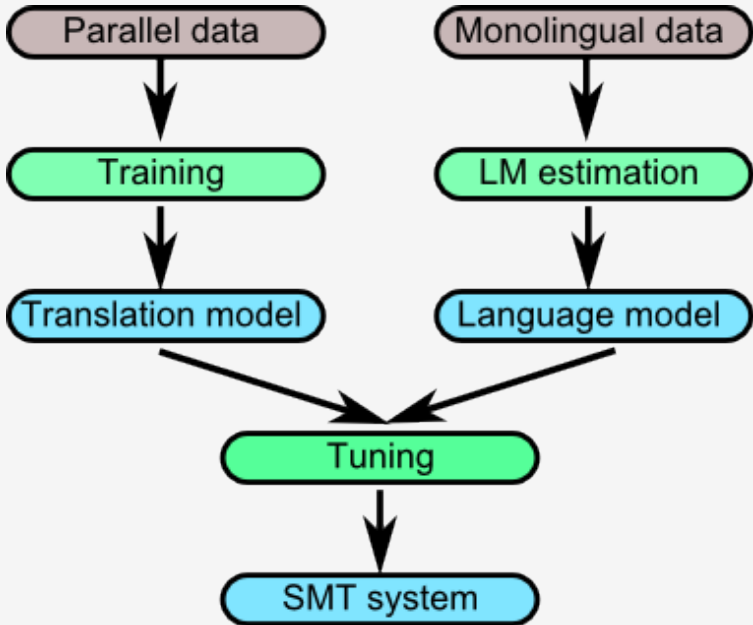
**Artem Sokolov**

Computerlinguistik

Universität Heidelberg

Sommersemester 2015

material from P. Koehn, R. Shapire, D. McAllester, Chen & Goodman



- the simplest way to estimate LM – MLE
- is problematic on sparse data:
  - ➔ problem with zero counts
  - ➔  $\Rightarrow$  zero probabilities for unseen n-grams
  - ➔ high perplexity on test
  - ➔  $\Rightarrow$  low performance of the model as a whole

$$p = \frac{c + \alpha}{n + \alpha v}, \quad \alpha < 1, \alpha \text{ optimized on held-out set}$$

$$r^* = \frac{T_r^1 + T_r^2}{N_r^1 + N_r^2}$$

$N_r$  - # of n-grams that occur  $r$  times

$$r^* = (r + 1) \frac{N_{r+1}}{N_r}$$

$N_r$  - # of n-grams that occur  $r$  times

$$p_n^I(w_i | w_{i-n+1}, \dots, w_{i-1}) = \lambda_{w_{i-n+1}, \dots, w_{i-1}} p_n(w_i | w_{i-n+1}, \dots, w_{i-1}) \\ + (1 - \lambda_{w_{i-n+1}, \dots, w_{i-1}}) p_{n-1}^I(w_i | w_{i-n+2}, \dots, w_{i-1})$$

$$p_n^{BO}(w_i | w_{i-n+1}, \dots, w_{i-1}) = \begin{cases} d_n(w_{i-n+1}, \dots, w_{i-1}) p_n(w_i | w_{i-n+1}, \dots, w_{i-1}) & \text{if } \text{count}_n(w_{i-n+1}, \dots, w_i) > k \\ \alpha_n(w_i | w_{i-n+1}, \dots, w_{i-1}) p_{n-1}^{BO}(w_i | w_{i-n+2}, \dots, w_{i-1}) & \text{otherwise} \end{cases}$$



**What is the probability of observing  $w$  “in the wild”, if we saw it with some frequency in the corpus?**

- $p(w)$
- sample  $S$  of size  $|S|$  from  $p(w)$
- $c(w)$  = the number of times  $w$  occurs in  $S$
- $S_r = \{w : c(w) = r\}$
- $M_r = \sum_{w \in S_r} p(w)$

$M_r$  is a useful quantity:

- If we know it,  $p(w)$  for  $w \in S_r$  is  $\frac{M_r}{|S_r|}$   
(total mass divided by total number of distinct elements)

**Example:** To see the need for a smoothing, imagine we have sampled a large  $S$  where each  $n$ -gram occurs exactly once (quite unlikely event). The naive way of estimating  $M_1$  would be

$$\frac{\# \text{ of times } w \text{ occurs in } S \times \# \text{ of different words we are ok with}}{\text{total size of } S} = \frac{k \times |S_1|}{|S|} = \frac{1 \times |S|}{|S|} = 1.$$

However, for any reasonable distribution  $p(w)$  the probability  $M_1$ , given such an unlikely sample  $S$ , should be close to 0 .

Find the expectation of  $M_r$ :

$$\mathbb{E}[M_r] = \sum_w p(w) P[w \in S_r] = \frac{r+1}{|S| - r} \mathbb{E}[|S_{r+1}|] - \frac{r+1}{|S| - r} \mathbb{E}[M_{r+1}]$$

Almost unbiased estimate of  $M_r$

$$\frac{r+1}{|S| - r} \mathbb{E}[|S_{r+1}|] \simeq \frac{r+1}{|S|} |S_{r+1}|.$$

**Final formula**

$$r^* = (r+1) \frac{|S_{r+1}|}{|S_r|}$$

## **"spite" , "constant"**

both occur 993 times in the Europarl

- 9 words follow "spite"; almost always followed by "of" (979 times)
- 415 words follow "constant"; "and" (42), "concern" (27), "pressure" (26) and singletons (268)

Much more likely to see a new bigram that starts with "constant" than with "spite".

WB-smoothing is an instance of the recursive interpolation:

$$p_{WB}(w_i | w_{i-n+1}^{i-1}) = \lambda_{w_{i-n+1}^{i-1}} p_{ML}(w^i | w_{i-n+1}^{i-1}) + (1 - \lambda_{w_{i-n+1}^{i-1}}) p_{WB}(w_i | w_{i-n+2}^{i-1})$$

Intuition:

- in back-offs, we back-off to lower-order is higher-order is missing
- interpret  $(1 - \lambda_{w_{i-n+1}^{i-1}})$  as the probability of recurring to the lower-order model
- use the number of unique words that follow the history to estimate this likeliness

- define the number of possible extensions of a history  $w_1, \dots, w_{n-1}$ :

$$N_{1+}(w_1, \dots, w_{n-1}, \bullet) = |\{w_n : c(w_1, \dots, w_{n-1}, w_n) > 0\}|$$

- Define

$$1 - \lambda_{w_1, \dots, w_{n-1}} = \frac{N_{1+}(w_1, \dots, w_{n-1}, \bullet)}{N_{1+}(w_1, \dots, w_{n-1}, \bullet) + \sum_{w_n} c(w_1, \dots, w_{n-1}, w_n)}$$

$$\begin{aligned}1 - \lambda_{spite} &= \frac{N_{1+}(\text{spite}, \bullet)}{N_{1+}(\text{spite}, \bullet) + \sum_{w_n} c(\text{spite}, w_n)} \\ &= \frac{9}{9 + 993} = 0.00898\end{aligned}$$

$$\begin{aligned}1 - \lambda_{constant} &= \frac{N_{1+}(\text{constant}, \bullet)}{N_{1+}(\text{constant}, \bullet) + \sum_{w_n} c(\text{constant}, w_n)} \\ &= \frac{415}{415 + 993} = 0.29474\end{aligned}$$

**Observation:**

Discount value  $1 - d_r$  in the GT smoothing are often “almost constant” (for  $r \gg 1$ ).

**Idea:**

Jelinek-Mercer interpolation with  $\lambda_{w_{i-n+1}^{i-1}} p(w_i | w_{i-n+1}^{i-1})$  set to

$$\frac{\max\{c(w_{i-n+1}^i) - D, 0\}}{\sum_{w_i} c(w_{i-n+1}^i)}.$$

**Final formula**

$$\hat{p}(w_i | w_{i-n+1}^{i-1}) = \frac{\max(c(w_{i-n+1}^i) - D, 0)}{c(w_{i-n+1}^i)} + \frac{DN_{1+(w_{i-n+1}^{i-1} \bullet)}}{\sum_{w_i} c(w_{i-n+1}^i)} \hat{p}(w_i | w_{i-n+2}^{i-1})$$



Consider the word "York" (477 times). As frequent as the words "foods", "indicates" or "providers".

In a unigram LM, will have a respectable probability.

However, it almost always directly follows "New" (473 times).

## Problem

- unigram model is used, if the bigram model is inconclusive.
- "York" is unlikely to be the second word in an unseen bigram
- therefore "York" **should have a low probability.**

**Idea:**

set the unigram probability to the number of different words that it follows instead of number of occurrences

**Formalize:**

$$\begin{aligned} \frac{c(w_i)}{\sum_{w_i} c(w_i)} &= \sum_{w_{i-1}} \hat{p}(w_{i-1}w_i) = \sum_{w_{i-1}} \hat{p}(w_i|w_{i-1})p(w_{i-1}) \\ &= \sum_{w_{i-1}} \hat{p}(w_i|w_{i-1}) \frac{c(w_{i-1})}{\sum_{w_{i-1}} c(w_{i-1})}. \end{aligned}$$

For absolute discounting we had:

$$\hat{p}(w_i|w_{i-n+1}^{i-1}) = \frac{\max(c(w_{i-n+1}^i) - D, 0)}{c(w_{i-n+1}^i)} + \frac{DN_{1+(w_{i-n+1}^{i-1} \bullet)}}{\sum_{w_i} c(w_{i-n+1}^i)} \hat{p}(w_i|w_{i-n+2}^{i-1})$$

Substitute into the constraint:

$$c(w_i) = c(w_i) - N_{1+}(\bullet w_i)D + D\hat{p}(w_i)N_{1+}(\bullet\bullet),$$

where

$$N_{1+}(\bullet w_i) = |\{w_{i-1} : c(w_{i-1}w_i) > 0\}|,$$

$$N_{1+}(\bullet\bullet) = |\{(w_{i-1}, w_i) : c(w_{i-1}w_i) > 0\}|.$$

$$\hat{p}(w_i) = \frac{N_{1+}(\bullet w_i)}{N_{1+}(\bullet\bullet)}.$$

**Idea**

Use 3 discount factors  $D_1, D_2, D_{3+}$

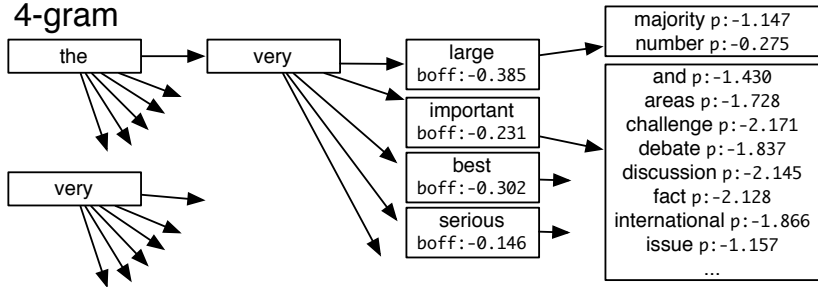
$$D = D(c) = D_1 \mathbf{1}[c = 1] + D_2 \mathbf{1}[c = 2] + D_{3+} \mathbf{1}[c > 2].$$

Perplexity for language models trained on the Europarl corpus:

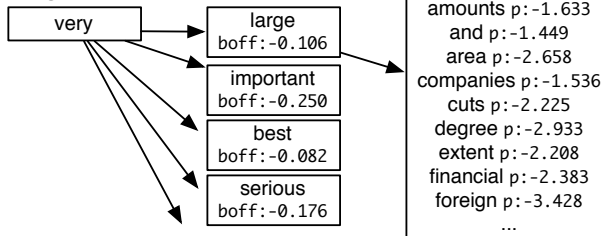
<b>Smoothing method</b>	<b>bigram</b>	<b>trigram</b>	<b>4-gram</b>
Good-Turing	96.2	62.9	59.9
Witten-Bell	97.1	63.8	60.4
Modified Kneser-Ney	95.4	61.6	58.6
Interpolated Modified Kneser-Ney	94.5	59.3	54.0

- estimation on disk
- efficient structures (trie)
  - ➔ 'the very large majority'
  - ➔ 'the very large number'
  - ➔ shared history

## 4-gram



## 3-gram backoff



Backoff from 4-gram to 3-gram:

$$\begin{aligned} p_{\text{LM}}(\text{amount}|\text{the very large}) &= \text{backoff}(\text{the very large}) \\ &\quad \cdot p_3(\text{amount}|\text{very large}) \\ &= \exp(-0.385 + -2.510) \end{aligned}$$



- estimation on disk
- efficient structures (trie)
  - ➔ 'the very large majority'
  - ➔ 'the very large number'
  - ➔ shared history
- fewer bits to store numbers (num. indexes/huffman)
- bin probabilities
- reduce vocabulary (dates/numbers)
- filtering irrelevant n-grams

