Learning to Search

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15 October 2018

Learning to Search

structured prediction as search problem

A structured learning problem consists of

- an input space $\mathcal{X}$
- an output space $\mathcal{Y}$
- a fixed but unknown data distribution $\mathcal{D}$ over $\mathcal{X} \times \mathcal{Y}$
- a loss function $\ell(y^*, \hat{y}) \to \mathbb{R}^+$, which measures the distance between the true (y^*) and predicted $(\hat{y})$ outputs.

The goal of structured learning is to use N samples, $\{x_i, y_i\}_{i=1}^N \sim \mathcal{D}$ and learn a mapping $f : \mathcal{X} \to \mathcal{Y}$,
that minimizes the expected loss $\mathbb{E}_{(x,y)\sim\mathcal{D}}[\ell(y, f(x))]$

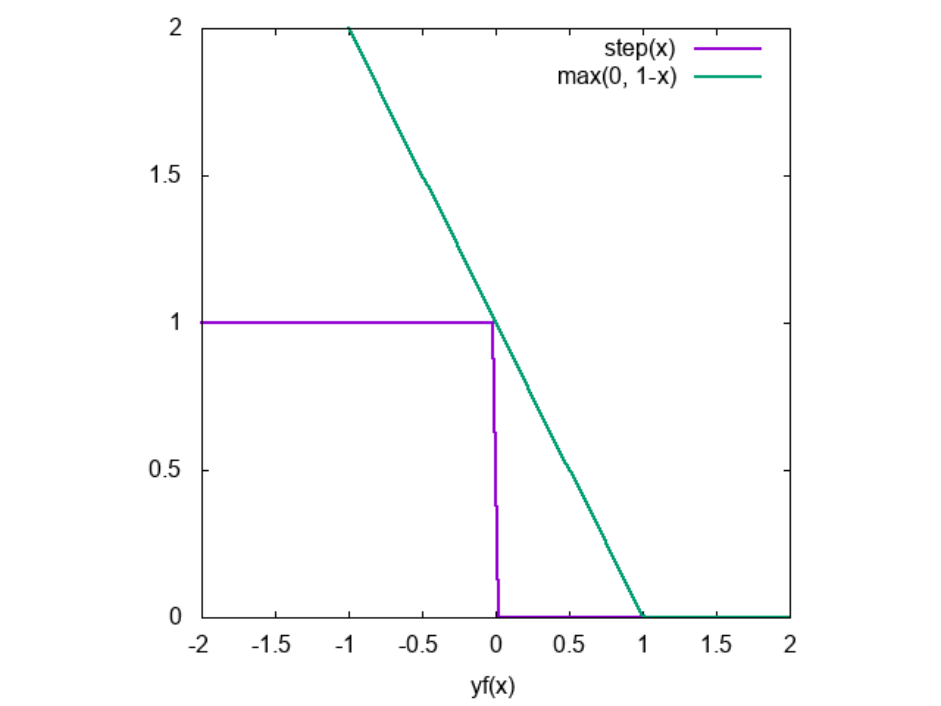
Perceptron [Rosenblatt, 1958]

init: $w_0 = 0$
for $t = 0, \ldots$ **do**
    observe x_t
    predict $\hat{y}_t = sign(w_t^\top x_t)$ (values: 0 or 1)
    recieve true label y_t
    if $\hat{y}_t \neq y_t$ **then**
        update $w_{t+1} = w_t + x_t$
    else
        keep $w_{t+1} = w_t$
    end if

- the $0/1$ loss, $\mathbb{I}[y \cdot w^\top x \leq 0]$, is unfortunately non-convex
- we already know that we should build some convex surrogate:

$$\ell(w, x) = \max(0, 1 - y \cdot w^\top x)$$

- this is called 'hinge-loss'
- it's convex and upper-bounds the $0/1$ loss, $\ell_t(w, x) \geq \mathbb{I}[y \cdot w^\top x \leq 0], \forall w$



Now we have an online convex optimization task:

1. receive x_t
2. predict $p_t = sign(w_t^\top x_t)$
3. suffer loss $\ell_t(w, x_t) = \max(0, 1 - y \cdot w^\top x_t)$
4. update w_t

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We know already Follow-The-Regularized-Leader as a way to no-regret:

$$w_t = \arg\min \sum_{i=1}^{t} \ell_t(w, x_i) + \frac{1}{2\nu} ||w||_2^2$$

We also know that for convex functions it is sufficient to work with linearized losses, setting $z_t = \nabla_w \ell_t(w)$:

Theorem

Consider FTRL, linear losses $\ell_t(w) = w^\top z_t$, and regularization $R(w) = \frac{1}{2\nu} ||w||_2^2$ and $w, u \in S = \mathbb{R}^d$, then

$$R_T(u) \leq \frac{1}{2\nu} ||u||_2^2 + \frac{\nu}{2} \sum_{t=1}^{T} ||z_t||_2^2 .$$

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- when there is no error the gradient is zero
- z_t is the gradient
- gradient of $w^\top x_t$ is just x_t

$$R_T(u) \leq \frac{1}{2\nu} ||u||_2^2 + \frac{\nu}{2} \sum_{t \in \{errors\}} ||x_t||^2 .$$

- and let $R = \max ||x_t||$ (bounded features)

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- use the definition of regret and the surrogate upper-bounding property

$$\#errors \leq \sum_t \ell(w_t) - \sum_t \ell(u) \leq \frac{1}{2\nu} ||u||_2^2 + \frac{\nu}{2} R^2 \cdot \#errors$$

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- set $\nu = \frac{||u||}{R\sqrt{\#errors}}$ and rearrange

$$\#errors - R\,||u|| \sqrt{\#errors} - \sum_{t=1}^{T} \ell_t(u) \leq 0$$

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- let's assume realizability, $\exists u^*$, s.t. $y_t = sign((u^*)^\top x_t), \forall t$

$$\#errors - R ||u^*|| \sqrt{\#errors} \leq 0$$

- if we additionaly assume, that $||u^*|| \leq 1/\gamma$ we get

$$\#errors \leq R^2/\gamma^2$$

[Novikoff, 1962]

Structured Perceptron [Collins and Roark, 2004]

input: training data $\{(x_t, y_t)\}_{t=1}^{N}$
init: $w_0 = 0$
for $t = 0, \ldots$ **do**
    observe x_t, y_t
    predict $\hat{y}_t = \arg\max_{y \in \mathcal{Y}(x)} w_t^\top \phi(x_t, y)$
    if $\hat{y}_t \neq y_t$ **then**
        update $w_{t+1} = w_t + \phi(x_t, y_t) - \phi(x_t, \hat{y}_t)$
    end if

Thm

If $||\phi(x_t, y_t)|| \leq R$ and $\exists u, ||u|| = 1$ such that $\forall t, y \in \mathcal{Y}(x_t)$:
$u^\top \phi(x_t, y_t) \geq u^\top \phi(x_t, y) + \gamma$, then

$$\# \text{ errors} \leq \frac{R^2}{\gamma^2}$$

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    if $\hat{y}_t \neq y_t$ **then**
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Let's look again at the structured SVM model from the previous lecture

StructSVM Objective

$$R(w) = \frac{1}{N} \sum_{i=1}^{N} \left(\max_{\mu} (w^\top F \cdot \mu + l_i^\top \mu) - w^\top F_i \cdot \mu_i \right) + \frac{\lambda}{2} ||w||^2$$

Rewriting in the same notation as perceptron:

StructSVM Objective

$$R(w) = \frac{1}{N} \sum_{i=1}^{N} \left(\max_{y \in \mathcal{Y}} (w^\top \phi(x_t, y) + \ell(y_t, y)) - w^\top \phi(x_t, y_t) \right) + \frac{\lambda}{2} ||w||^2$$

Rewriting in the same notation as perceptron:

StructSVM Objective

$$R(w) = \frac{1}{N}\sum_{i=1}^{N}\Big(\max_{y\in\mathcal{Y}}(w^\top\phi(x_t,y)+\ell(y_t,y)) - w^\top\phi(x_t,y_t)\Big) + \frac{\lambda}{2}\,||w||^2$$

- $R(w)$ is convex (sum of affine & convex functions)
- denote $\tilde{y} = \arg\max_y(w^\top\phi(x_t,y)+\ell(y_t,y))$
- can be minimized using batch subgradient method

$$\frac{\partial R}{\partial w} = \frac{1}{N}\sum_{i=1}^{N}\Big(\phi(x_t,y_t) - \phi(x_t,\hat{y})\Big) + \lambda w$$

$$w_{t+1} = w_t - \alpha_t\frac{\partial R}{\partial w}$$

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Where do we get $\tilde{y}$?

$$\tilde{y}_i = \arg\min_{y \in \mathcal{Y}} (w^\top \phi(x_i, y) - \ell(x_i, y))$$

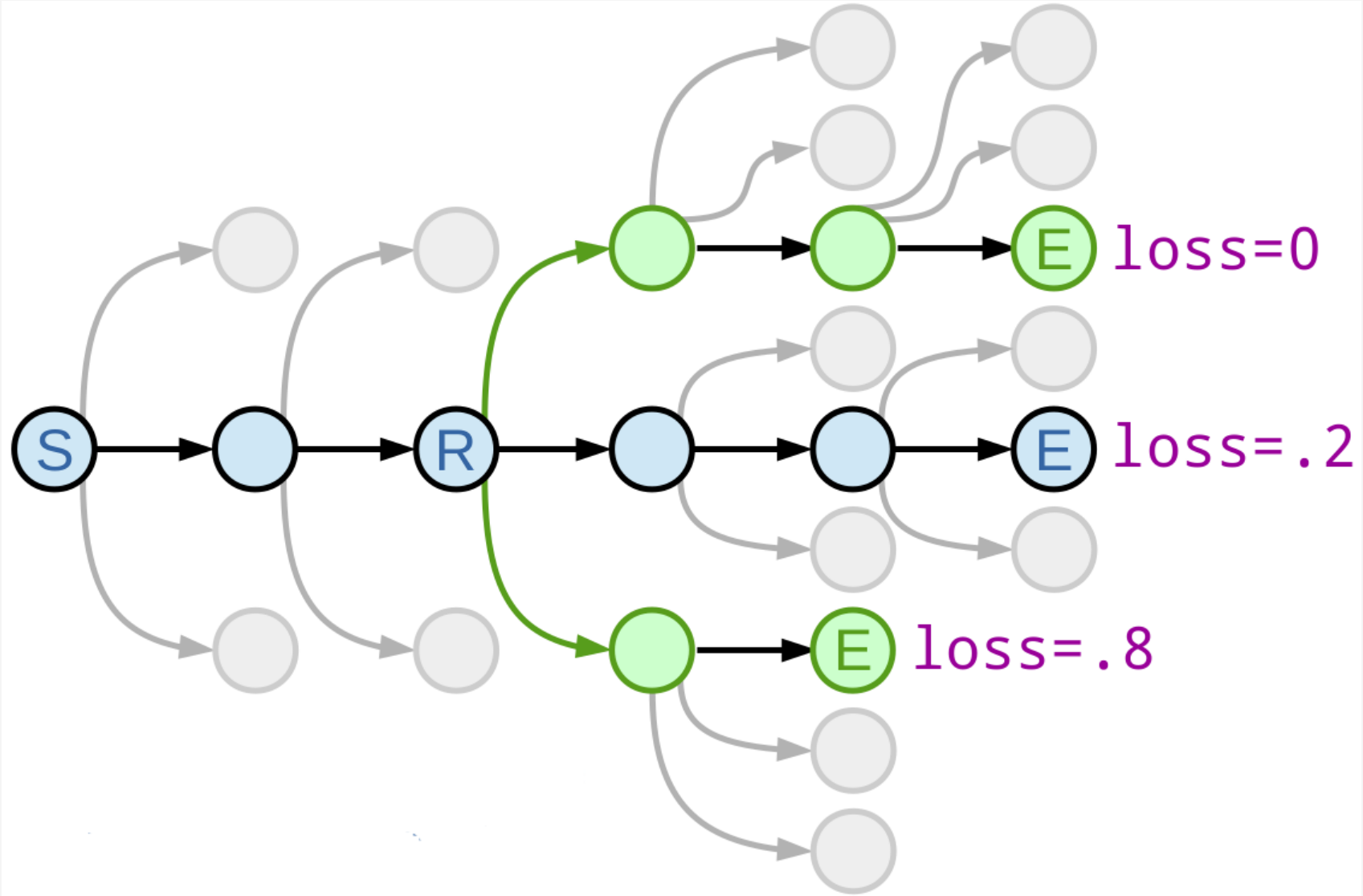
- this is the loss-augmented inference task
- requires a solution to a search problem in the underlying space
- has to be solved for every input instance (big data is a problem)
  - ➡ solvable in $O(T \cdot K)$ on chains and trees (long sequences are a problem)
  - ➡ much harder or intractable for general graphs (e.g. MRFs)

So we have two problems:

- when there is relation to the inference, the learning loop is expensive
  - ➡ the non-probabilistic models above (Perceptron or SVM)
- or inference and learning maybe not related at all
  - ➡ e.g. CRF: the learning objective knows nothing about inference
  - ➡ same for many deep learning-based models
- often finding $\tilde{y}$ alone is not enough
  - ➡ CRFs require feature expectations
  - ➡ large-margin methods require n-best lists
- Learning as Search Optimization (LaSo) is tackling these problems
  - get rid of the expensive $\arg\max$ (global decision)
  - decompose the structure building into local decisions

Setup:

- input $x \in \mathcal{X}$ induces a search space $\mathcal{Y}(x)$
- initial state b (also encodes x)
- set of states $\mathcal{S}$
- transition function $P(s_{t+1}|s_t, a_t)$ (here deterministic)
- for each (valid) sequence of states and actions, there is a corresponding output $y(e)$
- loss $\ell(e) = \ell(y^*, y(e))$, where y^* is the ground truth structure
- feature generating function $\phi : \mathcal{S} \to \mathbb{R}^d$, that expresses both the input x and previous actions
- agent follows a policy $\pi(a_t|s_t)$, which chooses an action a_t in state s_t
- trajectory $\tau_\pi = (a_1, s_1, \ldots, s_{T-1}, a_T, s_T)$, where $P(s_{t+1}|s_t, \pi(s_t))$
- T is maximum lengths of τ
- goal: $\min_\pi \mathbb{E}_{(x,y)\sim\mathcal{D}}[\ell(y, \tau_\pi))]$



[Daume'15]

- for any $s \in \mathcal{S}$ and $y \in \mathcal{Y}$ we can say if s can lead to y
- in this case we call it 'y-good', otherwise 'y-bad'
- goal:
  - ➡ first node can lead to any structure
  - ➡ the queue always contains at least one y-good node

Algo Learn($problem$, $initial$, $enqueue$, $\boldsymbol{w}$, x, y)
$nodes \leftarrow$ MakeQueue(MakeNode($problem$,$initial$))
while $nodes$ is not empty **do**
  $node \leftarrow$ RemoveFront($nodes$)
  if none of $nodes \cup \{node\}$ is y-good **or**
    GoalTest($node$) and $node$ is not y-good **then**
    $sibs \leftarrow siblings(node, y)$
    $\boldsymbol{w} \leftarrow update(\boldsymbol{w}, x, sibs, node \cup nodes)$
    $nodes \leftarrow$ MakeQueue($sibs$)
  else
    if GoalTest($node$) **then return** $\boldsymbol{w}$
    $next \leftarrow$ Operators($node$)
    $nodes \leftarrow enqueue(problem, nodes, next, \boldsymbol{w})$
  end if
end while

- online learning algorithm
- search and learning are tightly intervened
- usually needs a size restriction on the queue
- similar to structured perceptron, except for early updates
- updates are done on errors (if the current beam cannot lead to the right answer)

$$w_{t+1} = w_t + \sum_{n \in sibs} \frac{\phi(x, n)}{|sibs|} - \sum_{n \in nodes} \frac{\phi(x, n)}{|nodes|}$$

Analysis:

- basically, follows the analysis for structured perceptron
- result: for separable problems with margin γ, number of errors is $\leq \frac{R^2}{\gamma^2}$
- actually it's not exactly true for subtle reasons, see [Xu and Fern, 2007]

- natural only for sequence labelling problems
- hard to apply for tasks with production in arbitrary order

Literature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