# Significance

## Significance Testing under Measurement Variations

- State-of-the-art: Statistical significance testing is mostly ignored in NLP and ML in general. [Marie et al., 2021, Ulmer et al., 2022]
- Goal: Start reproducibility analysis by significance testing, w/ and w/o incorporation of variability in meta-parameters and data.
- Method:
  - Train LMEM on performance scores of baseline and SOTA models, obtained w/ or w/o meta-parameter variation during training.
  - Apply **GLRT** to system effect parameter of LMEM.
  - Analyze significance w/ and w/o meta-parameter variation and conditional on data properties.

### Reminder: General Form of LMEMs

For given dataset of N input-output pairs {(x<sup>n</sup>, y<sup>n</sup>)}<sup>N</sup><sub>n=1</sub>, general form of an LMEM is

$$\mathsf{Y} = \mathsf{X}\boldsymbol{\beta} + \mathsf{Z}\mathsf{b} + \boldsymbol{\epsilon}.$$

- Y are N stacked response variables,
- X and Z known design matrices,
- $\beta$  fixed effects,
- b random effects,
- $\epsilon$  residual errors,
- where  $b \sim \mathcal{N}(0, \psi_{\theta}), \ \epsilon \sim \mathcal{N}(0, \Lambda_{\theta}).$

#### GLRTs based on LMEMS

- Response variables Y for LMEM training: Performance evaluation scores of meta-parameter variants of baseline and SOTA systems.
- GLRT: Train LMEM with fixed effect β<sub>c</sub> accounting for competing systems on performance scores of baseline and SOTA systems, and compare their likelihood ratio.
- Pairing of systems on the sentence level: Incorporation of random sentence effect b<sub>s</sub> allows incorporation of meta-parameter variations and reduces residual variance.

The nested models setup [Pinheiro and Bates, 2000]

Restricted null hypothesis model not distinguishing between systems:

$$m_0: Y = \beta + b_s + \epsilon_{res},$$

where  $\beta$  is fixed effect for common mean for both systems,  $b_s$  is random effect for sentence-specific deviation with variance  $\sigma_s^2$ , and residual error  $\epsilon_{res}$  with variance  $\sigma_{res}^2$ .

**General model with different means** for baseline and SOTA:

$$m_1: Y = \beta + \beta_c \cdot \mathbb{I}_c + b_s + \epsilon_{res},$$

where indicator function  $\mathbb{I}_c$  activates fixed effect  $\beta_c$  for deviation of competing SOTA model from the baseline mean  $\beta$  when data point was obtained by a SOTA evaluation.

#### GLRTs in the nested models setup

- Restricted model m<sub>0</sub> is special case ("nested") of more general model m<sub>1</sub> since it restricts factor β<sub>c</sub> to zero.
- Let  $\ell_0$  be likelihood of restricted model  $m_0$ ,  $\ell_1$  be likelihood of more general model  $m_1$ , intuition of GLRT is to reject the null hypothesis if the **test statistic of likelihood ratio**

$$\lambda = \frac{\ell_o}{\ell_1}$$

yields values close to zero.

#### Analyzing significance conditional on data properties

- Extend models m<sub>0</sub> and m<sub>1</sub> by a fixed effect β<sub>d</sub> modeling a test data property d like segment length, readability, or word rarity.
- Add interaction effect  $\beta_{c:d}$  to assess expected system performance for different levels of d.
- Perform GLRT comparing

$$m'_1: Y = \beta + \beta_d \cdot d + (\beta_c + \beta_{c:d} \cdot d) \cdot \mathbb{I}_c + b_s + \epsilon_{res}$$

to null hypothesis model

$$m'_0: Y = \beta + \beta_d \cdot d + b_s + \epsilon_{res}.$$

### Example: Interactive Machine Translation

Fine-Tuning Neural Machine Translation (NMT) from human feedback [Kreutzer et al., 2020]

- Baseline: NMT system pre-trained on large out-of-domain data.
- SOTA: Fine-tuning on in-domain data annotated with human error markings or error corrections.
- Response variables for LMEM training: TER scores on test data.

Data properties: Sentence lengths, binned into short (< 15 words), typical (15 – 55 words), very long (> 55 words).

<sup>[</sup>Snover et al., 2006]

#### Example: Interactive Machine Translation

Meta-parameter	Grid values			
learning_rate	0.0001	0.0003	0.0005	0.003
random_seed	42	43	44	
encoder_dropout	0	0.2	0.4	0.6
decoder_dropout	0	0.2	0.4	0.6
decoder_dropout_hidden	0	0.2	0.4	0.6

Meta-parameter grid of attention-based RNN for interactive NMT.

[Kreutzer et al., 2020]

# Significance Testing of Difference Baseline - SOTA



- TER scores for fine-tuning on human error markings or human post-edits compared to baseline, evaluated on test sentences of 3 length classes.
  - SOTA systems trained under three different random seeds, thus one replication for each of three random seeds in LMEM input data.

# Significance Testing of Difference Baseline - SOTA

	<i>p</i> -value	effect size	
baseline - marking	0.000332	1.24	
baseline - post-edit	0.000000358	1.28	
marking - post-edit	0.0252	0.589	

- *p*-values and effect sizes (standardized mean difference) for comparison of fine-tuning on human error markings or human post-edits to baseline on very long test sentences.
  - *p*-values < 0.05, medium to very large effect sizes

## Significance Testing under Meta-Parameter Variation



Extended meta-parameter configuration space by grid search over  $4 \times 4 \times 4 \times 4 \times 3 = 768$  trained models for each of the fine-tuning runs.

# Significance Testing under Meta-Parameter Variation

#### Meta-parameters:

- initial learning rate (learning\_rate),
- probability of zeroing out connections during training of encoder (enc\_dropout), decoder (dec\_dropout), and hidden decoder layers (dec\_dropout\_h),
- seed of random number generator (random\_seed).
- *p*-values for all pairwise differences are above 0.05 across different classes of sentence length.
  - Significance of result difference lost!
  - Investigate reasons → reliability analysis!

#### Advantages of Model-Based Significance Testing with LMEMs

- One-stop approach to test statistical significance of performance differences between machine learning models:
  - Variance in evaluation scores due meta-parameter variation is incorporated naturally into training data for LMEM.
  - No matching of evaluation metrics to significance tests required [Dror et al., 2020] since test statistics is not based on evaluation metrics, but on MLE parameters of LMEM [van der Vaart, 1998].
  - Further key advantage is analysis of significance of result difference conditional on data properties.
  - Power of significance test is intimately related to reliability of model under analysis - next chapter!
  - Further reading: [van der Vaart, 1998, Pinheiro and Bates, 2000, Davison, 2003].

## Alternative: Sampling-Based Significance Tests

#### Goal:

- Applicability to arbitrary and arbitrarily complex evaluation metrics (e.g., non-linear combinations of counts like F-score [Manning et al., 2008], BLEU [Papineni et al., 2002], ROUGE [Lin and Hovy, 2003]).
- No restriction to "mean of samples" metrics which is requirement in parametric tests (*t*-test, *Z*-test).
- More powerful than nonparametric tests (e.g. sign test).

#### Examples

- Bootstrap resampling: [Efron and Tibshirani, 1993] Sample itself is a representative "proxy" for the population, sampling distribution of test statistic is estimated by repeatedly sampling (with replacement) from the sample itself.
- Permutation test: [Fisher, 1935] Principle of stratified shuffling
  [Noreen, 1989] allows generation of null-hypothesis conditions by shuffling
  (sampling without replacement) outputs between two systems at strata that partition the data.

Given test set outputs  $(A_0, B_0) = (a_i, b_i)_{i=1}^N$ , where  $a_i$  is the output of system  $\mathcal{A}_i$ , and  $b_i$  is the output of system  $\mathcal{B}_i$ , on test instance *i*. Compute score difference  $\Delta S_0 = S(A_0) - S(B_0)$  on test data. For k = 1, ..., K: Generate bootstrap dataset  $S_k = (A_k, B_k)$  by sampling N examples from  $(a_i, b_i)_{i=1}^N$  with replacement. Compute score difference  $\Delta S_k = S(A_k) - S(B_k)$  on bootstrap data. Compute  $\overline{\Delta S_k} = \frac{1}{K} \sum_{k=1}^{K} \Delta S_k$ . Set c = 0. For k = 1, ..., K: If  $|\Delta S_k - \overline{\Delta S_k}| \geq |\Delta S_0|$ c + +p = c/K. Reject null hypothesis if p is less than or equal to rejection level  $\alpha$ .

Given test set outputs  $(A_0, B_0) = (a_i, b_i)_{i=1}^N$ , where the first element in the ordered pair  $(a_i, b_i)$  is the output of system  $\mathcal{A}$ , and the second element is the output of system  $\mathcal{B}_{i}$ , on test instance *i*. Compute score difference  $\Delta S_0 = S(A_0) - S(B_0)$  on test data. Set c = 0. For r = 1, ..., R: Compute shuffled outputs  $(A_r, B_r)$  where for each i = 1, ..., N:  $\operatorname{swap}(a_i, b_i) = \begin{cases} (a_i, b_i) & \text{with probability 0.5,} \\ (b_i, a_i) & \text{with probability 0.5.} \end{cases}$ Compute score difference  $\Delta S_r = S(A_r) - S(B_r)$  on shuffled data. If  $|\Delta S_r| \geq |\Delta S_0|$ c + +p = c/R. Reject null hypothesis if p is less than or equal to rejection level  $\alpha$ .

- Bootstrap test makes more Type I errors (i.e., rejecting H<sub>0</sub> when it is true) and more Type II errors (i.e., not rejecting H<sub>0</sub> when it is false) than the permutation test if **bootstrap consistency** is not given (i.e., if data from which is resampled are not representative of population). [Canty et al., 2006, Riezler and Maxwell, 2005, Berg-Kirkpatrick et al., 2012]
- Designed for comparing a pair of selected systems on a single test set, no easy incorporation of variability in meta-parameters or data!

- Permutation test has great power (i.e., high probability of rejecting  $H_0$  when it is false) for large samples [Hoeffding, 1952].
- Stratified shuffling principle needs to be applicable, which is not always the case.
- Designed for comparing a pair of selected systems on a single test set, no easy incorporation of variability in meta-parameters or data!

Significance testing across multiple meta-parameter and data settings

- Bootstrap and permutation tests are designed for comparing a pair of selected systems on a single test set - extensions apply this principle to sampling w/ and w/o replacement from system outputs under meta-parameter variations, but ignore variation of data properties. [Clark et al., 2011, Sellam et al., 2021, Bouthillier et al., 2021].
- Statistical significance test based on the stochastic order/dominance of performance score distributions allow incorporation of meta-parameter variation, but still ignore variation of data properties. [Dror et al., 2019, Ulmer et al., 2022]